ECONOMICS HARBOUR

IIT-JAM ECONOMICS STUDY MATERIAL

(FIRST EDITION)

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UNIT- 1

MICRO-ECONOMICS

IIT-JAM Economics Study Material
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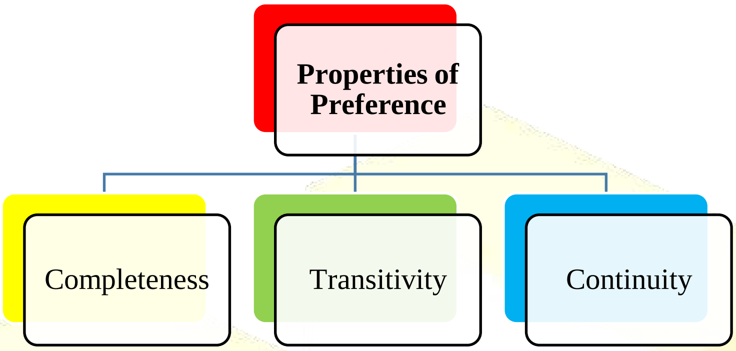
CONSUMER THEORY

PREFERENCE AND UTILITY

Axioms of Rational Choice:

The preference relation is assumed to have three basic properties:



1. Completeness: If A and B are any two situations, the individual can always specify exactly one of the following three possibilities:

a. A is preferred to B.
b. B is preferred to A.
c. A and B are equally attractive.

Under this, people are assumed not to be paralysed by indecision. The assumption also rules out the possibility that an individual can report both that A is preferred to B and that B is preferred to A.

2. Transitivity: If an individual reports that 'A is preferred to B' and 'B is preferred to C', then he or she must also report that 'A is preferred to C'. This assumption states that the individual's choices are internally consistent.

3. Continuity: If an individual reports 'A is preferred to B', then situations suitably 'close to' A must also be preferred to B. The purpose of the assumption is to rule out certain kinds of discontinuous, knife-edge preferences that pose problems for choice.

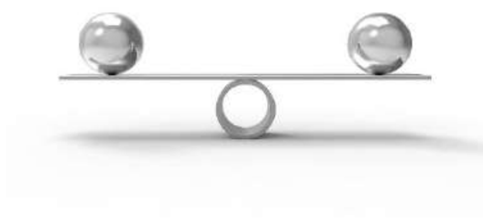
Utility

- **Jeremy Bentham** gave the term 'utility'.
- According to Bentham, more desirable situations offer more utility than do less desirable ones, that is, if a person prefers situation A to B, it would imply that utility from A (U(A)) exceeds the utility from B (U(B)).

  1. Non-uniqueness of utility measures:
  - ✓ Notion of utility is defined only upto an order-preserving (monotonic) transformation.
  - ✓ This lack of uniqueness in the assignment of utility numbers also implies that it is not possible to compare utilities of different people.

2. The ceteris-paribus assumption:

In the case of utility, a common practice is to devote attention exclusively to choices among quantifiable options while holding constant, the other things that affect behaviour.



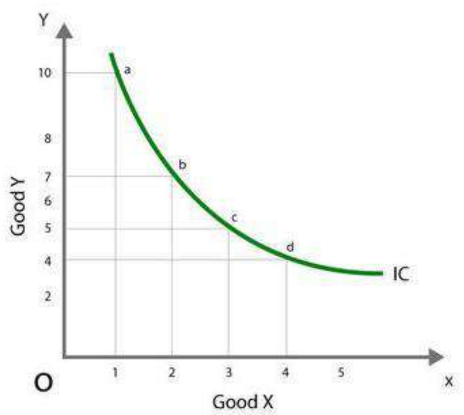
3. Utility from consumption of goods:

Utility = U (x,y)

Other factors kept constant.

Indifference Curve and Marginal Rate of Substitution

Indifference curve shows a set of consumption bundles about which the individual is indifferent, that is, the bundles all provide the same level of utility.



Marginal Rate of Substitution: The negative slope of an indifference curve at some point is termed the marginal rate of substitution (MRS) at that point.

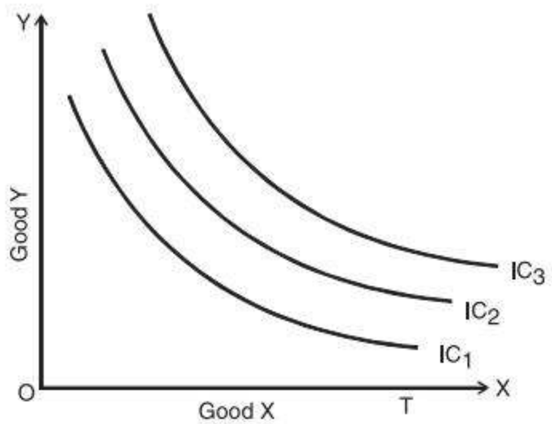
MRS = - dy/dx

Marginal rate of substitution represents the rate at which the individual is willing to trade x for y while remaining equally well-off.

Indifference curve is drawn on the assumption of a diminishing marginal rate of substitution.

Indifference curve map

Indifference curve map is a bundle of several indifference curves.

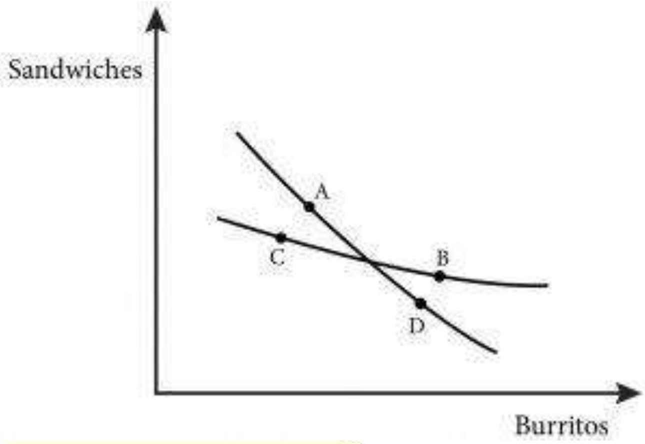


The utility increases with a higher or rightward movement of indifference curve. This is because of the assumption that more of a good is preferred to less.

Indifference curve and transitivity

Say there are four bundles – A, B, C, D

By the assumption of non-satiation, 'A is preferred to B' and 'C is preferred to D'. But B and C lie on the same indifference curve. So the axiom of transitivity implies that A must be preferred to D. But this is not true as A and D lie on the same indifference curve. Therefore, **indifference curves cannot intersect.**



Mathematics for Indifference Curves

$$MRS = \frac{dy}{dx} = -\frac{MU_x}{MU_y}$$

This is the rate at which x can be traded for y and is given by the negative ratio of the marginal utility of good x to that of good y. This is because increases in quantity of good x must be met by decreases in the quantity of good y to keep utility constant.

Utility Functions for Specific Preferences

1. Cobb-Douglas Utility:

$$U(x,y) = x^{\alpha}y^{\beta}$$

Assuming $\alpha+\beta = 1$,

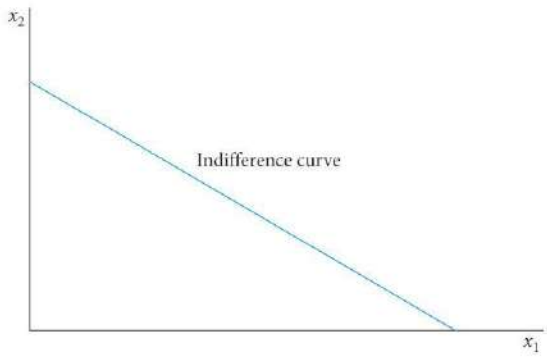
$$U(x,y) = x^{\delta}y^{1-\delta}$$

Where: $\delta = \alpha/(\alpha+\beta)$ and $1-\delta = \beta/(\alpha+\beta)$

2. Perfect Substitutes

$$U(x,y) = \alpha x + \beta y$$

Where α, β are positive constants.

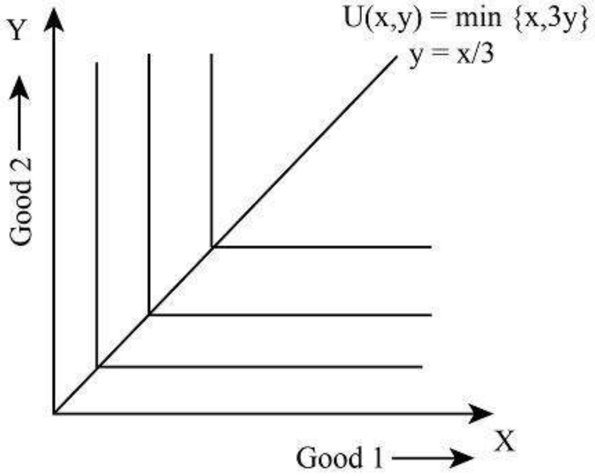


Indifference curves are straight lines as marginal rate of substitution is constant and equal to α/β.

This implies that a person with these preferences would be willing to give up the same amount of x_2 to get one more x_1, no matter how much of it was consumed.

3. Perfect Complements:

$$U(x,y) = \min(\alpha x, \beta y)$$



Consumption will occur at vertices of the indifference curves.

4. Constant Elasticity of Substitution (CES) Utility:

$$U(x,y) = [x^{\delta} + y^{\delta}]^{1/\delta}$$

If δ = 1, x and y are perfect substitutes

If δ approaches zero, the function approaches Cobb-Douglas.

If δ approaches (-)∞, the function approaches the case of perfect complements.

$$\textit{Elasticity of substitution } (\sigma) = \frac{1}{(1-\delta)}$$

σ = Infinity	Perfect substitutes
σ = Zero	Perfect complements

Homothetic Preferences

In consumer theory, a consumer's preferences are called homothetic if they can be represented by a utility function which is homogenous of degree one.

Marginal rate of substitution depends only on the

REPRESENTATION THEOREM

If a consumer has a preference relation, that is, complete, reflexive, transitive, strongly monotonic and continuous, then these preferences can be represented by a continuous utility function U(x) such that U(x) > U(x') if and only if x > x'.

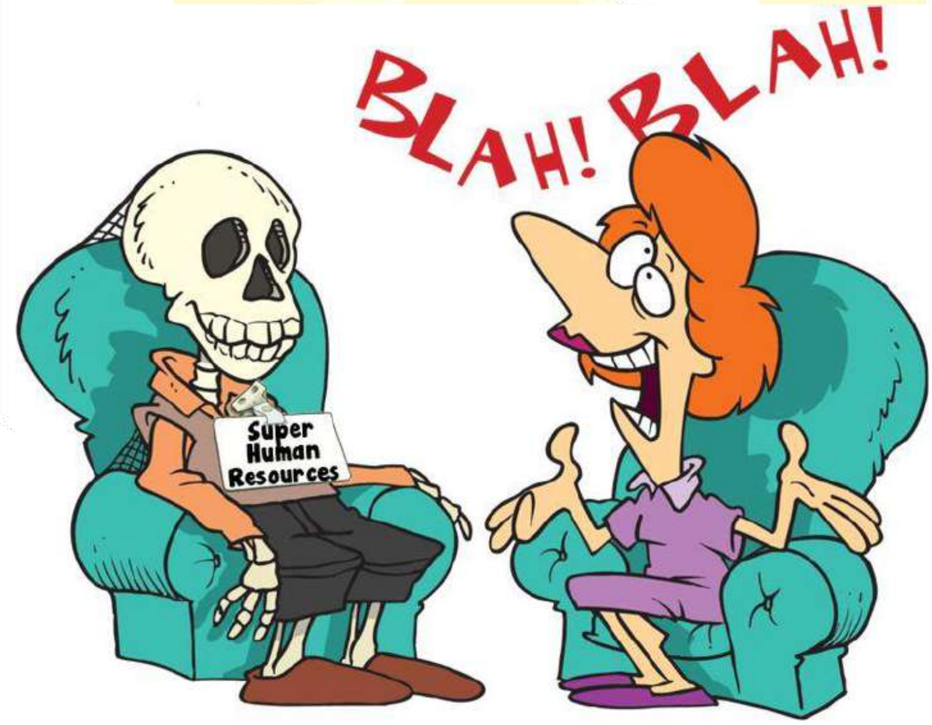
Proof:

Let e = (1,1,…, 1). For each x, define U(x) by U(x)e~x. Then U(x) is a utility function for the preferences if

1. Such a function U(x) exists.
2. The function U(x) is unique.
3. U(x) > U(x') if and only if x > x'.

Let B = {a:ae≥x}. If x = (x_1, x_2), let y = {max(x_1,x_2), max(x_1,x_2)}. Then strong monotonicity implies that y > x. So B is not empty. Let W = {a: x≥ae}. Then zero is an element of W, so W is not empty. By completeness, B U W = {a: a≥0}. Both B and W are closed sets, so B ∩ W is not empty. Therefore, there is some 'a' such that ae~x. By strong monotonicity, if a' > a, then a'e > x, and if a' < a, then x > a'e, so 'a' is unique.

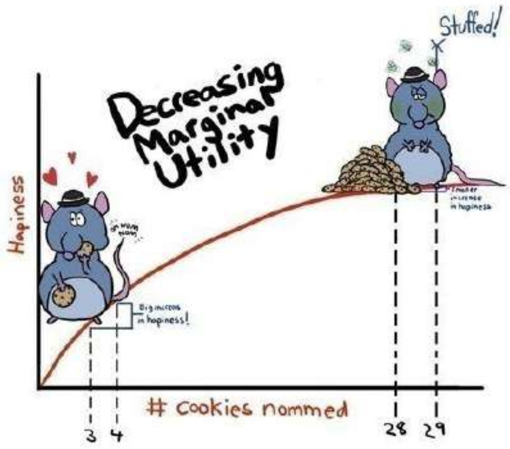
Let U(x) = a. So U(x) exists and it is unique. Next, we want to show that U(x) represents the preferences. Suppose that x and y are two consumption levels and U(x) = a_x where a_xe~x. Let U(y) = a_y where a_ye~y. If a_x > a_y then by monotonicity a_xe > a_ye. By transitivity, x~a_xe > a_ee~y. Finally, if x > y then a_xe > a_ye so that a_x > a_y.



UTILITY MAXIMISATION AND CHOICE

Utility Maximisation

To maximise utility, given a fixed amount of income to spend, an individual will buy those quantities of goods that exhaust his or her total income and for which the rate of trade-off between any two goods (MRS) is equal to the rate at which the goods can be traded one for the other in the market place.

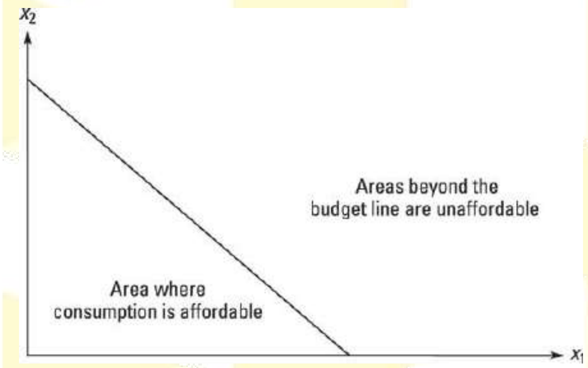


Budget Constraint

Assume that an individual has income, I to allocate between good x and good y. If p_x is the price of good x and p_y is the price of good y, then the individual is constrained by

$$p_x x + p_y y \leq I$$

Graphically,



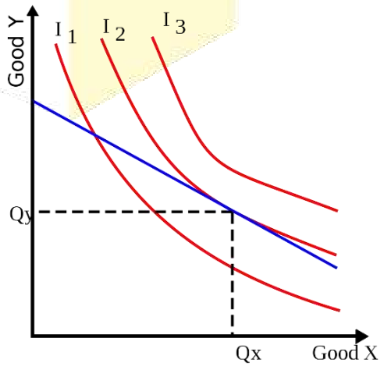
First-Order Conditions for Maximisation

Slope of budget constraint = Slope of Indifference curve

$$-\frac{p_x}{p_y} = MRS_{xy}$$

$$-\frac{p_x}{p_y} = \frac{dy}{dx} = MRS_{xy}$$

Graphically,



This is a necessary condition for maximisation.

Second-Order Conditions for Maximisation

If MRS is assumed to be always diminishing (IC is convex to the origin), in that case, condition of tangency is both a necessary and sufficient condition for a maximum.

Therefore, the second order condition states that indifference curves must always be convex to the origin.

The n-Good Case

U = U (x1, x2, …, xn)

Subject to

$I = p_1x_1 + p_2x_2 + \ldots + p_nx_n$

$I - p_1x_1 - p_2x_2 - p_nx_n$

Lagrangian expression:

$$L = U(x_1, x_2, \ldots, x_n) + \lambda(I - p_1x_1 - p_2x_2 - p_nx_n)$$

$$\frac{dL}{dx_1} = \frac{dU}{dx_1} - \lambda p_1 = 0$$

$$\frac{dL}{dx_2} = \frac{dU}{dx_2} - \lambda p_2 = 0$$

.

.

$$\frac{dL}{dx_n} = \frac{dU}{dx_n} - \lambda p_n = 0$$

$$\frac{dL}{d\lambda} = I - p_1x_1 - p_2x_2 - \cdots - p_nx_n = 0$$

First-Order condition:

$$\frac{dU}{dx_i} \div \frac{dU}{dx_j} = \frac{p_i}{p_j}$$

$$MRS = \frac{p_i}{p_j}$$

Interpretation of Lagrange Multiplier

$$\lambda = \frac{dU/dx1}{p_1} = \frac{dU/dx2}{p_2} = \cdots = \frac{dU/dxn}{p_n}$$

These equations state that, at the utility-maximising point, each good purchased should yield the same marginal utility per rupee spent on that good.

Example:

Suppose in a Cobb-Douglas production function, x sells for $1 and y sells for $4 and that total income is $8. Also $\alpha = \beta = 0.5$. Calculate the quantities of x and y consumed and total utility. (A = 1)

$U = x^{\alpha}y^{\beta}$

$U = x^{0.5}y^{0.5}$

$L = x^{\alpha}y^{\beta} + \lambda(I - p_x x - p_y y)$

$dL/dx = \alpha x^{\alpha-1}y^{\beta} - \lambda p_x = 0$

$dL/dy = \beta x^{\alpha}y^{\beta-1} - \lambda p_y = 0$

$dL/d\lambda = I - p_x x - p_y y = 0$

$$\frac{\alpha x^{\alpha-1}y^{\beta}}{p_x} = \lambda = \frac{\beta x^{\alpha}y^{\beta-1}}{p_y}$$

$$p_y y = \frac{\beta}{\alpha}p_x x = \frac{1-\alpha}{\alpha}p_x x$$

Where: $\beta+\alpha = 1$, $\beta = 1-\alpha$

$I = p_x x + p_y y$

$I = p_x x + [(1-\alpha)/\alpha]p_x x$

$I = p_x x\, [1+(1-\alpha)/\alpha]$

$I = (1/\alpha)p_x x$

$x^* = \alpha I/p_x$

Similarly,

$y^* = \beta I/p_y$

Substituting the values,

$x^* = 4$

$y^* = 1$

$U = 2$

Indirect Utility Function

$$X_1^* = X_1(p_1, p_2, \ldots, p_n, I)$$

$$X_2^* = X_2(p_1, p_2, \ldots, p_n, I)$$

$$\ldots$$

$$X_n^* = X_n(p_1, p_2, \ldots, p_n, I)$$

$$\text{Max } U = U\,[\,X_1^*(p_1, p_2, \ldots, p_n, I), X_2^*(p_1, p_2, \ldots, p_n, I), \ldots, X_n^*(p_1, p_2, \ldots, p_n, I)]$$

$$\text{Max } U = V(p_1, p_2, \ldots, p_n, I)$$

Due to the individual's desire to maximise utility given a budget constraint, the optimum level of utility obtainable will depend indirectly on the prices of goods being bought and the

UNIT- 2

MACRO-ECONOMICS

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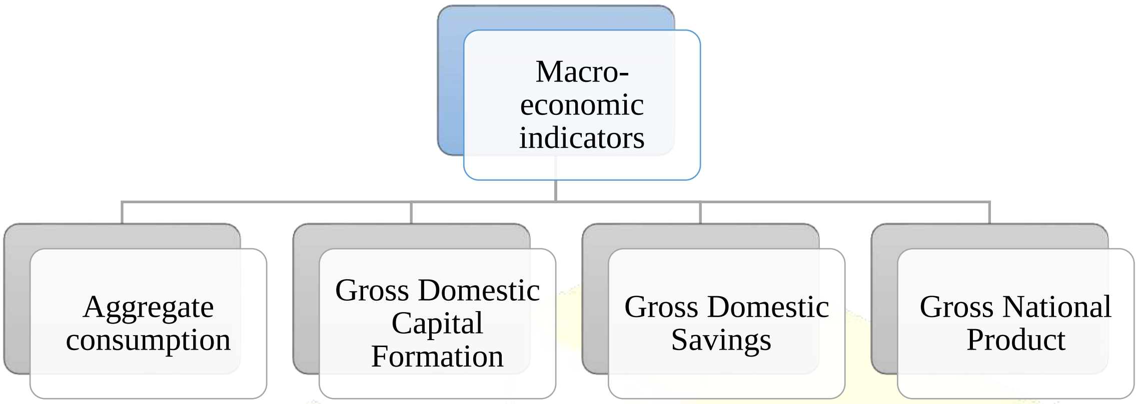
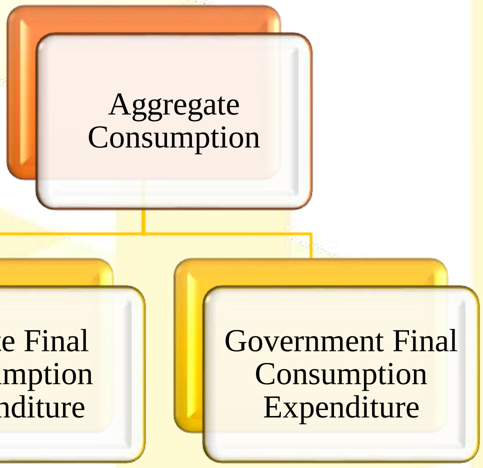
NATIONAL INCOME ACCOUNTING

Important macro-economic indicators are:



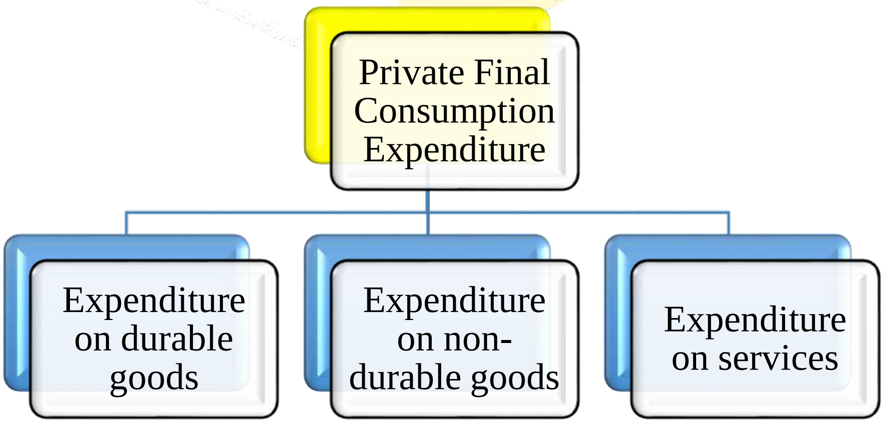
1. Aggregate Consumption:

Reflects on the standard of living of the people in a country.



a. Private Final Consumption Expenditure: Money expenditure made by resident households and non-profit institutions on purchases of goods and services for satisfaction of their wants.

- Includes value of goods produced by farmers for their self-consumption.
- Includes imputed value of rent of self-occupied houses by the households.
- Includes wages and salaries in kind and money value of gifts received by the households.



b) Government's Final Consumption Expenditure: The collective services such as defence, law and order, healthcare provided by the government to the households.

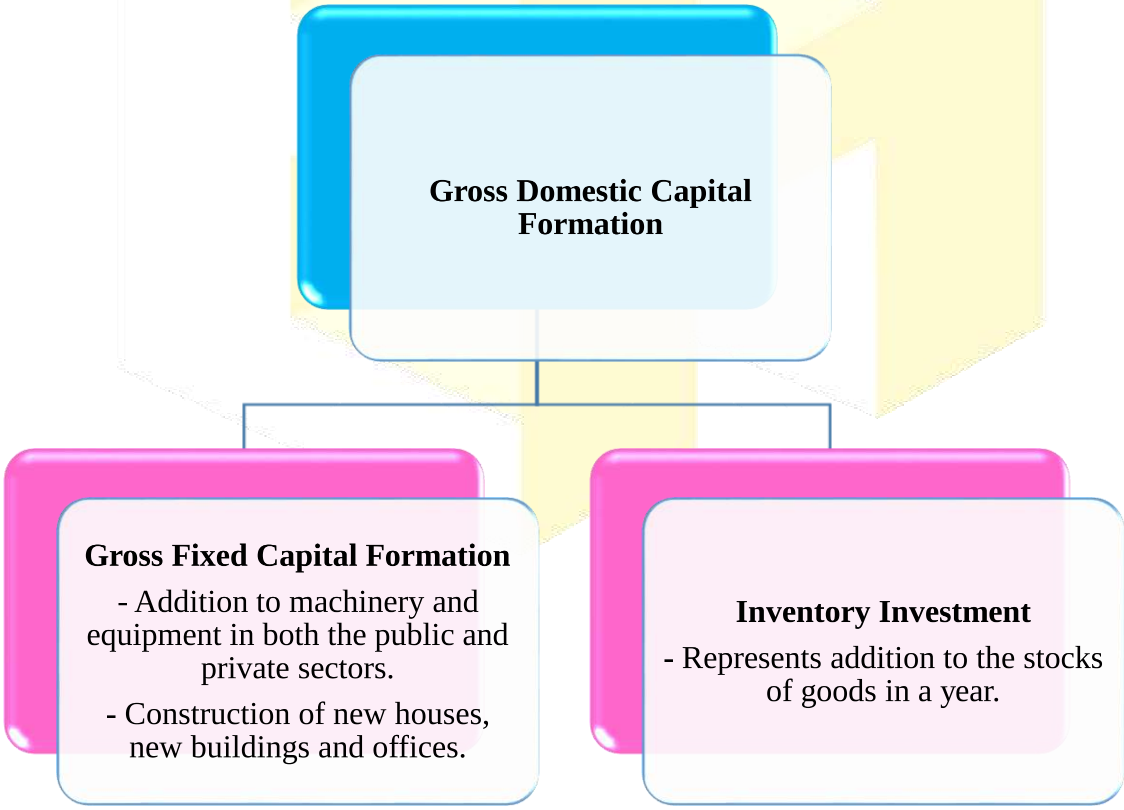
Includes:

- Value of goods and services purchased by the government for use by its employees for providing collective services and these represent intermediate consumption.
- Wages and salaries
- Costs incurred by the government on account of consumption of fixed capital for providing the services.

Note: Governments also make expenditure on transfer payments. Against these transfer payments, government does not get any services in exchange in the current period. They are, therefore, not counted in estimation of national income. Besides, the recipients of these transfer payments will either spend them on their consumption or use them for investment. If we count these transfer payments, it would lead to double counting.

2. Gross Domestic Capital Formation (GDCF):

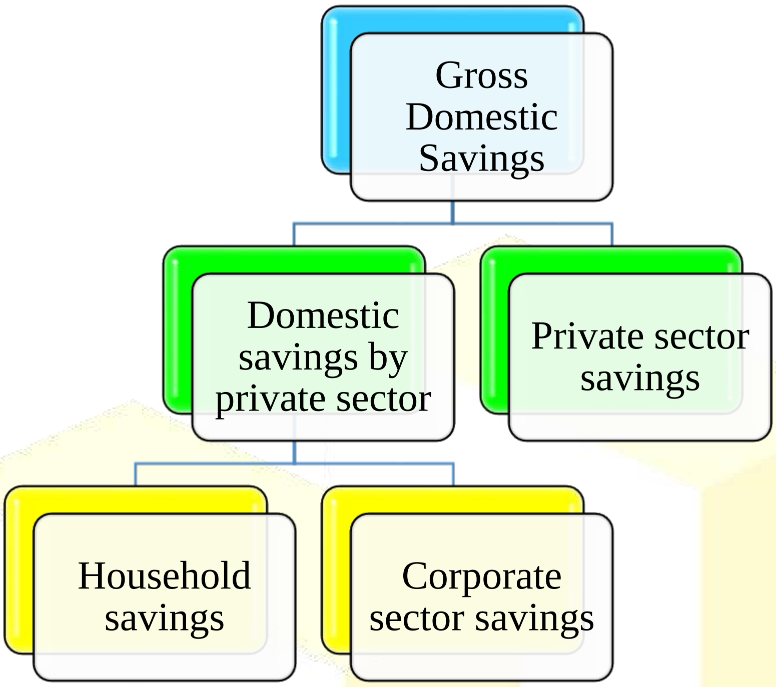
Also referred to as aggregate investment, GDCF represents the addition to the stock of physical capital goods such as factories, machine plants and equipment which are meant for use in future production of goods and services.



GDCF = Gross Fixed Capital Formation + Inventory Investment

3. Gross Domestic Savings

According to Central Statistical Organisation (CSO):



4. Gross Domestic Product (GDP):

Growth of GDP: Supply side

Factors are as follows:

a) Primary sector: Consists of agriculture, forestry and fishing

b) Industrial sector: Consists of

i) Mining and quarrying

ii) Manufacturing

iii) Electricity, gas and water supply

c) Services:

i) Construction

ii) Trade and hotels

iii) Transport, storage and communications

iv) Financing, insurance, real estate and business services

Growth of GDP: Demand side

Factors involved are as follows:

a) Private final consumption expenditure

b) Government final consumption expenditure

c) Investment expenditure

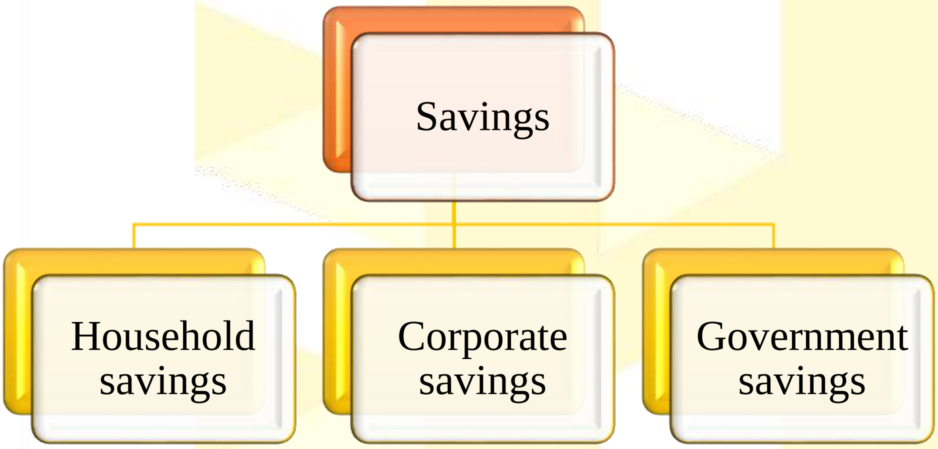
d) Net exports

Role of Capital Formation

- Capital goods are man-made instruments of production and increase the productive capacity of the economy.
- Accumulation of capital every year greatly increases the national product or income.
- Capital-accumulation also helps in achieving large-scale production and specialisation. The advantage of large-scale production and specialisation is that they greatly increase output and productivity and thereby bring down the cost of production per unit.
- Capital formation makes the technological progress of the economy possible.
- Capital-formation creates employment.

Determinants of Capital Formation

1. Rate of Savings:



a) Household savings: Factors that affect household savings are:

i) Level of per capita income and distribution of income:

- Greater the level of per capita income, greater will be the amount of saving.
- Greater the inequalities of income, greater will be the amount of savings in the economy. If the national income is equally distributed, much of the income will be consumed and a little would be saved. On the contrary, if national income is unequally distributed, a larger amount of savings will be done by richer people.

ii) Interest rate:

The rise in interest rate brings about increase in supply of savings. The effect of rise in interest rate has income effect as well as substitution effect. A rise in interest rate increases income of the individual and makes him better off. This induces him to consume more in the present and thus tends to reduce savings. But the rise in interest rate also increases return on savings. This induces him to postpone consumption and therefore, tends to increase savings. The net effect of rise in interest rate is uncertain.

iii) Social security provisions: Social security measures adopted by the government lowers savings.

iv) Taxation system: Income tax discourages savings.

v) Bequest motive: If high inheritance taxes are levied, they would discourage savings.

vi) Precautionary motive: People save for unforeseen contingencies. N

vii) Rate of inflation: In the economy, when there is high inflation or prices are continuously rising, value of money declines. The rise in prices or the fall in the value of money has an adverse effect on the savings in the economy.

viii) Growth rate of per capita income: If income per capita is not growing, the task of raising the marginal rate of saving is likely to be difficult. If per capita income is rising due to increase in productivity, it becomes possible to save relatively more out of increment in income and thereby rate of saving can be increased while increasing absolute consumption at the same time.

b) Corporate Saving: Business enterprises save when they do not distribute whole of their profits, but retain a part of them in the form of undistributed profits.

c) Government Saving: Includes the surplus of revenue obtained through taxes and surpluses from public undertakings over and above the current expenditure of the government.

INDIA's GROWTH MIRACLE

From 2003-04 to 2007-08, India's average annual growth rate of GDP rose above 9% per annum. In 2008-09, while the advanced developed countries were experiencing recession, India succeeded in achieving 6.7% growth which further rose to 8.4% in 2009-10 and 2010-11. This was mainly due to increase in saving and investment.

NATIONAL INCOME AGGREGATES

1. Gross National Product (GNP):

Defined as the total market value of all final goods and services produced by normal residents of a country in a year.

a) Measures the market value of annual output.
b) Includes the market value of only final goods.
c) Includes value of goods and services produced in a particular year.
d) Value of goods and services produced by normal residents of a country.

Components of GNP:

a) Value of final consumption goods and services = C
b) Value of new capital goods produced and addition to the inventories of goods = I
c) Purchases of goods and services by the government = G
d) Net exports = Exports – Imports = X – M
e) Net Factor Income from Abroad (Difference between factor incomes received from abroad by normal residents of India and factor incomes paid to the foreign residents for factor services rendered by them in the domestic territory of India.

2. Gross Domestic Product (GDP):

Money value of all final goods and services produced by all normal residents working in the domestic territory of a country but does not include net factor income from abroad.

$$\mathbf{GDP_{MP} = GNP_{MP} - \text{Net factor income from abroad}}$$

$$\mathbf{GDP_{MP} = C + I + G + (X - M)}$$

$$\mathbf{GDP_{FC} = GDP_{MP} - \text{Indirect taxes} + \text{subsidies}}$$

3. Net National Product (NNP):

$$\mathbf{NNP = GNP - Depreciation}$$

4. National Income:

- Also known as NNP_{FC}
- Shows how much it costs the society in terms of economic resources to produce net output.

$$\mathbf{NNP_{FC} = NNP_{MP} - \text{Indirect taxes} + \text{Subsidies}}$$

5. Personal Income:

Sum of all incomes actually received by all individuals or households during a year.

Personal Income = National Income – Social Security Contributions – Corporate income taxes – Undistributed Corporate Profits + Transfer Payments

6. Personal Disposable Income (PDY):

After a part of personal income is paid to government in the form of personal taxes like income tax, personal property tax, etc. what remains of personal income is called disposable income.

PDY = Personal income – Personal taxes

Or

PDY = Consumption + Savings

MEASUREMENT OF NATIONAL INCOME

1. Value added method:

Also called output method or production method

Value added by all sectors (agriculture, industry, services)

-

Intermediate consumption like that of raw materials

-

Consumption of fixed capital

-

Net Indirect Taxes

=

NDP_{FC}

National Income = NDP_{FC} + Net factor income from abroad

Precautions:

a) Imputed rent values of self-occupied houses should be included.
b) Sale and purchase of second hand goods should not be included.
c) Value of production for self-consumption should be counted.
d) Value of services of housewives are not included.
e) Value of intermediate goods must not be counted.

2. Income method:

National income is obtained by summing up the incomes of all individuals of a country.

Factor payments include:

a) Compensation of employees which includes wages and salaries, employer's contribution to social security schemes.

b) Rent
c) Interest
d) Profits: Dividends, Undistributed profits, Corporate income tax
e) Mixed income of the self-employed

Adding above we get NDP_{FC}.

$$\textbf{National Income} = \textbf{NDP}_{FC} + \textbf{NFIA}$$

Precautions:

a) Transfer payments are not included.
b) Imputed rent of self-occupied houses are not included.
c) Illegal money is not included.
d) Windfall gains are not included.
e) Corporate profit tax should not be separately included.
f) Death duty, gift tax, wealth tax, tax on lotteries are not included.
g) Receipts from sale of second-hand goods are not included.
h) Value of production used for self-consumption is included.

3. Expenditure Method:

Add up:

a) Private Final Consumption Expenditure (C)
b) Government's Final Consumption Expenditure (G)
c) Gross Domestic Capital Formation (I)
d) Net exports (X-M)

$$GDP_{MP} = C + I + G + (X\text{-}M)$$

$$NDP_{MP} = GDP_{MP} - \text{Depreciation}$$

$$NDP_{FC} = NDP_{MP} - \text{Indirect taxes} + \text{Subsidies}$$

$$NNP_{FC} = NDP_{FC} + \text{Net Factor Income from Abroad}$$

Precautions:

a) Second hand goods not included.
b) Purchase of shares and bonds should not be included.
c) Expenditure on transfer payments should not be included.
d) Expenditure on intermediate goods should not be included.

Important Concepts

Inflation and Price Indices

Real GDP: Measures changes in physical output in the economy between different time periods by valuing all goods produced in the two periods at the same prices.

Nominal GDP: Measures the value of output in a given period in the prices of that period.

Nominal GDP changes from year to year for two reasons:

1. Physical output of goods changes
2. Market prices changes

Inflation and Prices: Inflation is the rate of change in prices and the price level is the accumulation of past inflations.

$$\textbf{Inflation} = \frac{P_t - P_{t-1}}{P_{t-1}}$$

Where:

P_t = Current price level

P_{t-1} = Price level last year

If inflation is negative, it is called deflation.

GDP Deflator

- Ratio of nominal GDP in a given year to real GDP of that year.
- Widely used to measure inflation.
- Measures the change in prices that have occurred between the base year and the current year.

Consumer Price Index

Measures the cost of buying a fixed basket of goods and services representative of the purchases of urban consumer.

Difference between Consumer Price Index and Deflator

Consumer Price Index	Deflator
Less number of goods.	More number of goods.
Measures the cost of a given basket of goods, which is the same from year to year.	Basket of goods included in GDP deflator differs from year to year.
Includes prices of imports.	Includes prices of goods produced within the nation.

Personal Consumption Expenditure Deflator

Measures inflation in consumer purchases based on the consumption sector of the national income accounts.

Producer Price Index (PPI)

- Measures the cost of a given basket of goods.
- Designed to measure prices at an early stage of the distribution system.
- PPI is constructed from prices at the level of the first significant commercial transaction.
- Relatively flexible price index.

Core Inflation

Prices of some goods are very volatile, suggesting that price changes are often temporary. For this reason, policy makers focus on core inflation, which excludes changes in food and energy prices.

The End

UNIT- 3

STATISTICS FOR ECONOMICS

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STATISTICS FOR ECONOMICS

PROBABILITY THEORY

- ✓ Probability of a given event is an expression of likelihood of occurrence of an event.
- ✓ Probability is the ratio of the number of favourable cases to the total number of equally likely cases.
  P(A) = Number of favourable cases/Total number of equally likely cases.

Important Terms

1. Experiment: An experiment is the process of making an observation or taking a measurement which results in different possible outcomes.

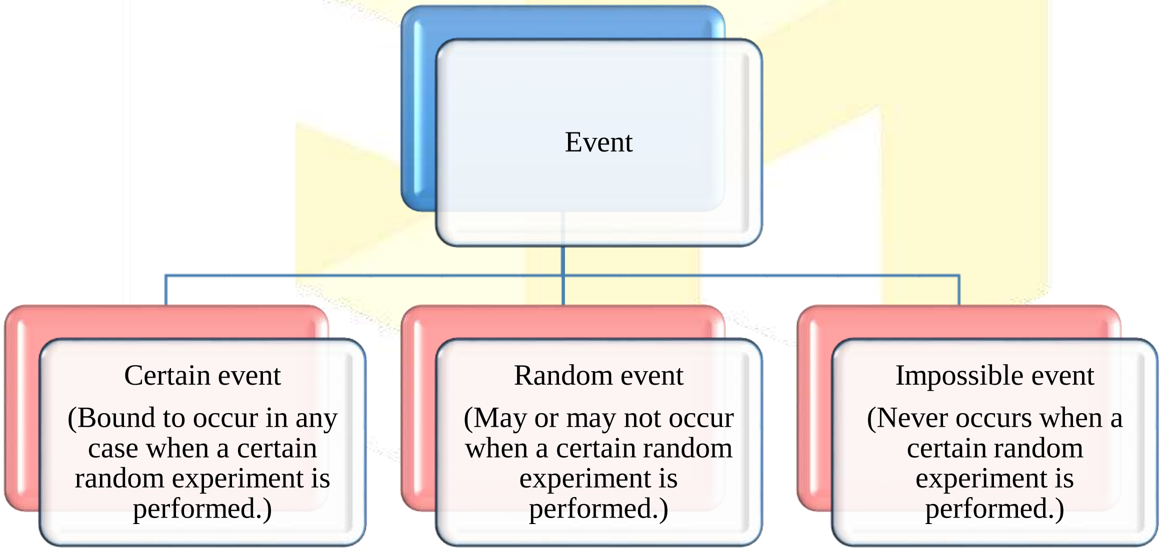
An experiment is called a random experiment if

a) all possible outcomes are known in advance.
b) None of the outcomes can be predicted with certainty.

2. Event: An event is the outcome of the experiment.

3. Equally likely: It implies that each outcome of an experiment has the same chance of appearing as any other.

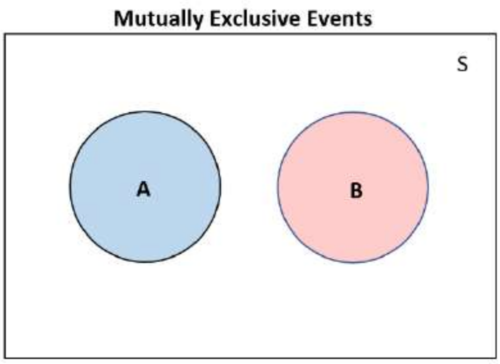
Events



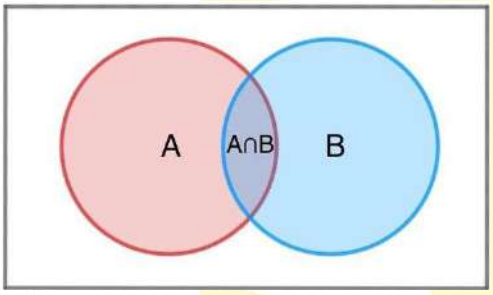
Types of Events

1. **Equally likely events:** Two or more events are said to be equally likely if any one of them cannot be expected to occur in preference to the others.
2. **Collectively Exhaustive Events:** Events are said to be collectively exhaustive when their totality includes all the possible outcomes of a random experiment.

3. **Complementary Events:** A is called a complementary event of B, if A and B are mutually exclusive and exhaustive.
4. **Simple Events:** An event is said to be simple when the occurrence of a single event is considered.
5. **Compound events:** An event is said to be compound when the joint occurrence of two or more events is considered.
6. **Mutually exclusive events:** Two or more events are said to be mutually exclusive if they cannot occur simultaneously in the same trial.



7. **Overlapping events:** Two or more events are said to be overlapping or intersection events if part of one event and part of another event can occur simultaneously in the same trial.
8.



9. **Independent events:** Two or more events are said to be independent if the occurrence of one does not affect the occurrence of the other.
10. **Dependent events:** Two or more events are said to be dependent if the occurrence of one affects the occurrence of the other.
11. **Mutually exclusive and exhaustive events:** A number of events are said to be mutually exclusive and exhaustive events if every two of them are mutually exclusive and one of them necessarily occurs in any trial.

Examples of Events:

Name of the Event	**Example**
Equally Likely Event	In a tossing of an unbiased coin, head is likely to come up as tail.
Collectively Exhaustive Events	In a tossing of an unbiased coin, possible outcomes are head or tail. Therefore, the total number of exhaustive cases are two.
Complementary Events	In a tossing of an unbiased coin, occurrence of head and tail.
Simple events	In a tossing of two unbiased coins, only one head occurs.
Compound events	In a tossing of two unbiased coins, one or more heads occur.

Mutually exclusive events	In a tossing of an unbiased coin, either head or tail appear.
Overlapping events	In a throw of dice, occurrence of odd or even numbers
Independent events	P(A ∩ B) = P(A).P(B) In a throw of dice, probability of getting a multiple of two is 3/6 or ½.
Dependent events	In a pack of cards, getting a heart is 13/52 and drawing of another heart is without replacing the card is 12/51.
Mutually Exclusive Event	In a pack of cards, drawing a spade and a heart are mutually exclusive events.

Axioms of Probability and their Properties

The axioms of probability are:

1. The probability of an event A in the sample space S is a non-negative real number:

$P(A) \geq 0$, for every event $A \subset S$.

2. The probability of a sample space S is one.

$P(S) = 1$

3. If $A_1, A_2, \ldots$ is

a) a sequence of mutually exclusive events, that is,

$A_i \cap A_j = \phi$, for $i \neq j$, and $i,j = 1, 2, \ldots$

b) such that $A = \bigcup_{i=1}^{\infty} A_i$, then:

$P(A) = P(\bigcup_{i=1}^{\infty} A_i) = \sum_{i=1}^{\infty} P(A_i)$

Properties of Probability Axioms

1. Probability of the empty set:

The probability of empty set is zero.

$P(\phi) = 0$

Suppose, if $A_1 = A_2 = A_3 = \ldots = \phi$

Then, $P(\phi) = P(\bigcup_{i=1}^{\infty} A_i) = \sum_{i=1}^{\infty} P(A_i) = \sum_{i=1}^{\infty} P(\emptyset)$

Which will hold true if right hand side is an infinite sum of zeros.

Therefore,

$\mathbf{P(\phi) = 0}$

2. The Addition Law of Probability:

If $A_1, A_2, \ldots$ are mutually exclusive events, then the probability of their union is the sum of their probabilities.

$P(\cup_{i=1}^{\infty} A_i) = \sum_{i=1}^{\infty} P(A_i)$

Suppose, $A_{n+1} = A_{n+2} = \ldots = \phi$, then, $\cup_{i=1}^{n} A_i = \cup_{i=1}^{\infty} A_i = \sum_{i=1}^{\infty} P(A_i) = \sum_{i=1}^{n} P(A_i) + \sum_{i=n+1}^{\infty} P(A_i)$

(Note: $\sum_{i=n+1}^{\infty} P(A_i) = 0$)

3. The Complement Rule:

If A is an event, then $P(A^c) = 1 – P(A)$

By definition, A and its complement A^c are such that:

$A \cup A^c = A$ and $A \cap A^c = \phi$

Therefore, from the addition law,

$P(S) = P(A \cup A^c) = P(A) + P(A^c)$

Also, we know that $P(S) = 1$,

Thus,

$1 = P(A) + P(A^c)$

4. The Monotonicity Rule:

For any two events A and B, such that $B \subset A$, we have,

$$P(A) \geq P(B)$$

Let us say,

$$A = B \cup (B^c \cap A)$$

Also,

$$B \cap (B^c \cap A) = \phi$$

So that,

$$P(A) = P\{ B \cup (B^c \cap A)\}$$

$$= P(B) + P(B^c \cap A)$$

Which implies, since $P(B^c \cap A) \geq 0$, then

$$P(A) \geq P(B)$$

5. Probability of the Union:

For any two events A and B, then

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Let us consider,

$$A \cup B = A \cup (A^c \cap B) \text{ and } A \cap (A^c \cap B) = \phi$$

Also we know,

$$A^c \cap B = B - (A \cap B)$$

As a result,

$$P(A \cup B) = P(A) + P(A^c \cap B)$$

$$= P(A) + P(B) - P(A \cap B)$$

6. Boole's Inequality:

For the events $A_1, A_2, \ldots, A_n$, $P(A_1 \cup A_2 \cup \ldots \cup A_n) \leq \sum_{i=1}^{n} P(A_i)$

Let us say, n = 2, then

$$P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2) \leq P(A_1) + P(A_2)$$

Since,

$$P(A_1 \cap A_2) \geq 0$$

CONDITIONAL PROBABILITY

Two events A and B are said to be dependent when B can occur only when A is known to have occurred and vice-versa. The probability associated with such an event is called the conditional probability and is denoted by P(A/B).

If two events A and B are dependent, then the conditional probability of B given A is:

$$P(B/A) = P(AB)/P(A) = P(A \cap B)/P(A)$$

Similarly,

$$P(A/B) = P(AB)/P(B)$$

General rule of multiplication,

$$P(A \text{ and } B) = P(B) * P(A/B)$$

$$P(A \text{ and } B) = P(A) * P(B/A)$$

For three events, A, B and C,

$$P(ABC) = P(A) * P(B/A) * P(C/AB)$$

Formula for calculating conditional probabilities

Case	**Formula**
P(B/A)	$P(A \cap B/P(A)$
P(A/B)	$P(A \cap B)/P(B)$
$P(\bar{B}/A)$	$P(A \cap \bar{B})/P(A) = [P(A) - P(A \cap B)]/P(A)$
$P(A/\bar{B})$	$P(A \cap \bar{B})/P(\bar{B}) = [P(A) - P(A \cap B)]/(1-P(B))$
$P(B/\bar{A})$	$P(\bar{A} \cap B)/P(\bar{A}) = [P(B) - P(A \cap B)]/[1-P(A)]$
$P(\bar{A}/B)$	$P(\bar{A} \cap B)/P(B) = [P(B) - P(A \cap B)]/P(B)$
$P(\bar{B}/\bar{A})$	$P(\bar{A} \cap \bar{B})/P(\bar{A}) = [1 - P(A \cup B)]/(1-P(A))$
$P(\bar{A}/\bar{B})$	$P(\bar{A} \cap \bar{B})/P(\bar{B}) = [1-P(A \cup B)]/(1-P(B))$

Complementary Relationships

P(B/A) and P($\bar{B}/A$)	P(B/A) = 1 - P($\bar{B}/A$)
P(A/B) and P($\bar{A}/B$)	P(A/B) = 1 - P($\bar{A}/B$)
P(A/$\bar{B}$) and P($\bar{A}/\bar{B}$)	P(A/$\bar{B}$) = 1 - P($\bar{A}/\bar{B}$)
P(B/$\bar{A}$) and P($\bar{B}/\bar{A}$)	P(B/$\bar{A}$) = 1 - P($\bar{B}/\bar{A}$)

TRY SOLVING THESE!

Question 1: If P(A) = 0.25, P(B) = 0.15 and P(A∪B) = 0.30

Find the values of:

1. P(A∩B)

2. P(A∩$\bar{B}$)

3. P($\bar{A}$ ∩ B)

4. P($\bar{A}$ ∩ $\bar{B}$)

5. P($\bar{A}$ ∪ $\bar{B}$)

Answers: 1. 0.10, 2. 0.15, 3. 0.05, 4. 0.70, 5. 0.90

BAYE'S THEOREM

Baye's Theorem is based on the formula for conditional probability. It answers the question, "What is the probability of the state value given the sample or experimental results?"

Posterior probabilities are the conditional probabilities that are calculated by revising the prior probabilities in the light of sample information. Probabilities before revision by Baye's rule are called priori (or prior probabilities) because they are determined before the sample information is taken into account.

Let $B_1, B_2, \ldots, B_n$ be mutually disjoint events, satisfying

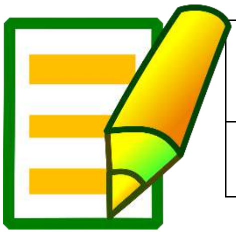
S = $\cup_{i=1}^{n} B_i$

And P (B_i) > 0 for every i= 1, 2, …, n. Then for every event A for which P(A) > 0, we have

$P(B_K/A) = [P(A/B_K)\ P(B_K)]/\sum_{i=1}^{n} P(\frac{A}{B_i})/P(B_i)$

RANDOM VARIABLES AND PROBABILITY DISTRIBUTION

- A random variable takes on different values as a result of the outcomes of a random experiment.
- It can either be discrete or continuous.



Discrete random variable	Can take up only certain values, often integers, but no intermediate values.
Continuous random variable	Allowed to assume any value within a given range.

Expected Value

- Expected value of a random variable is the sum of the products obtained by multiplying the various values that the variable can take by their corresponding probabilities.
- Usually denoted by E(X):

Discrete	$E(X) = p_1X_1 + p_2X_2 + \ldots + p_kX_k$
Continuous	$E(X) = \int_{-\infty}^{\infty} xf(x)dx$

- Expected value plays an important role in arriving at a decision under conditions of uncertainty when the probabilities of choice of different acts are known.
- The main disadvantage is that the model does not work in the absence of information on probabilities of states of nature.

Addition and Multiplication Laws

$$E(X+Y) = E(X) + E(Y)$$

$$E(XY) = E(X)*E(Y)$$

Variance:

$$E(X^2) - [E(X)]^2$$

Use of Geometric Progression Series in Probability

$$\boldsymbol{S_\infty = \frac{a}{1-r}}$$

a = First term, that is, player's probability of success in his first turn.

r = Common ratio, that is, product of probabilities of failure of all players.

Probability Distribution of a Random Variable

- Probability distribution of a random variable is a listing of the various values of random variable with their corresponding probabilities.

➢ Since a random variable has to assume one of its values, the sum of probabilities in a probability distribution must always be one.

EXAMPLE

A dealer in ready-made shirts has studied his record and notices that for the past 310 working days in the year, the demand for his shirts has varied as follows:

Demand ('000 units)	5	6	7	8	9	10
Number of days	20	60	80	120	20	10

What is the expected demand for his products?

Demand ('000 units)	5	6	7	8	9	10
Probability	20/310	60/310	80/310	120/310	20/310	10/310
Expected Demand	(20/310)*5	(60/310)*6	(80/310)*7	(120/310)*8	(20/310)*9	(10/310)*10

Total Expected Demand = 2260/310 = 7.3 thousand units

EXERCISE

Question 1: A dice is thrown. What is the probability of getting –

a. a multiple of 2 or 3.

b. a multiple of 2 or 4.

(Answer): a) 2/3 b) 1/2

Question 2: If P(A) = 0.5, P(B) = 0.3, P(AB) = 0.2, obtain the probability that

i) A occurs but not B

ii) At least one of A and B occurs

iii) Neither A nor B occurs

(Answer: i) 0.3, ii) 0.6, iii) 0.4

Question 3: In a city three daily newspapers X, Y, Z are published, 40 percent of the people of the city read X, 50 percent read Y, 30 percent read Z, 20 percent read both X and Y, 15 percent read X and Z, 10 percent read Y and Z and 8% read all the three papers. Calculate the percentage of people who do not read any of the three papers.

(Answer: 17%)

Question 4: A problem in statistics is given to five students, A, B, C, D and E. Their chances of solving it are 1/3, 1/5, 1/7, 1/8 and 1/9. What is the probability that the problem will be solved?

(Answer: 29/45)

Question 5: A and B are two events such that P(A) = 0.25, P(B) = 0.15 and P(B/A) = 0.40. Find the values of:

a) P (A∩B)

b) P(A/B)

c) P(A∪B)

(Answer: a) 0.10, b) 2/3, c) 0.30)

UNIT- 4

INDIAN ECONOMY

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INDIAN ECONOMY BEFORE 1950

The Indian economy, before 1950, was characterised as follows:

Agrarian Economy

- Majority of the population was engaged in agricultural activities.

-Traditional farming practices which were characterised by small holdings.

-Subsistence farming.

-Low productivity due to lack of modern agricultural techniques.

Colonial Exploitation

-Exploitation of natural resources, such as cotton, indigo, for export to Britain.

-Exploitative land revenue systems, such as Permanent Settlement, Ryotwari and Mahalwari Systems, extracted huge revenues from agricultural sector which led to the impoverishment of the peasants.

Limited industrialisation

-India's role was constrained to the provider of raw materials to promote British manufactured products and industries.

-Limited investment made in Indian industrial sector.

- Lack of modern factories and technologies.

- Few prevailing Indian industries were controlled by British interest.

Trade imbalances

-Unfavourable trade terms as India exported raw materials and imported British finished goods.

-This led to trade imbalances as imports exceeded exports for India.

Infrastructure Development

-The investment in Indian railways served British economic interests rather than contributing towards Indian economic development.

The role of railways was mainly to facilitate the transportation of raw materials from the hinterland to the ports for exports.

Famines and Economic Distress

-India witnessed several famines during the colonial period.

-Economic policies, combined with natural disasters, contributed to food shortages and economic distress, led to significant loss of life.

Currency System

-Indian currency system was often overvalued, making Indian exports cheaper for Britain but British imports more expensive for India.

Concentration of Wealth

-The economic policies during the colonial period led to the concentration of wealth in the hands of the few, the Indian elites and British.

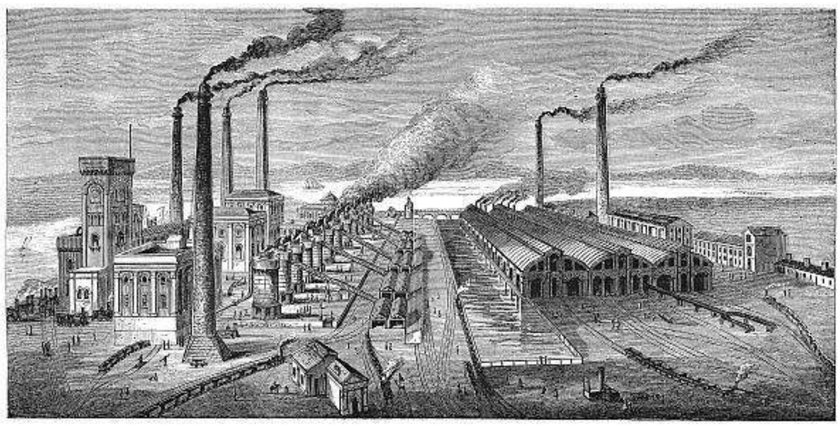
-This resulted into severe social and economic inequalities.

DE-INDUSTRIALISATION IN INDIA

- Before the colonial period, India's industrial sector was flourishing due to the traditional handicraft industries.
- However, the handicraft industries suffered a severe setback from 18th century onwards, due to British policies which made the industry struggle to survive.
- This period was considered as a phase of de-industrialisation for the Indian economy that resulted into the destruction of Indian handicraft industries due to competition from British manufactured products during the 19th century.

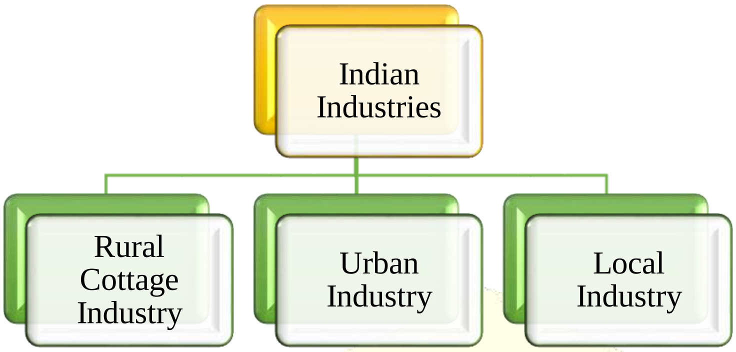
Year	India's share of manufacturing output
1800	19.7%
1860	8.6%
1913	1.4%

De-industrialisation is the process of decline in the proportion of working population engaged in secondary industry to total working population, or a decline in the proportion of population dependent on secondary industry to total population.



✓ ***In the case of India, handicraft industry succumbed before the machine-made goods.***

Classification of Indian Industries during Colonial Period



1. Rural Cottage Industry:

- ✓ Provided day-to-day requirements of the villagers.
- ✓ Included weavers, carpenters, potters, etc.
- ✓ Focussed on art and creativity.

2. Urban Industry:

- ✓ Primary industry was textile handicrafts, mainly cotton.

3. Local Industry:

- ✓ Neither rural nor urban due to the nature of raw materials used.
- ✓ For instance, iron industry which was concentrated in the region of central provinces.
- ✓ Method of production was crude and expensive.

Causes of De-industrialisation

1. Effect on cotton industries:

- ✓ The process of de-industrialisation destroyed the cotton industry.
- ✓ Indian cotton was widely popular before 1800.
- ✓ The Indian cotton industry was the largest provider of employment after agriculture.

2. Competition from machine-made goods:

- ✓ Due to industrial revolution of cotton textile industries in British, there was a steep rise in British imports in India and domination of British cloth in the Indian market, which resulted into the destruction of local industries.
- ✓ As a result, unemployment increased and the daily wages of spinners and weavers decreased.
- ✓ Other industries that suffered due to de-industrialisation were:
  a) Jute handloom weaving of Bengal

b) Woollen manufacturers of Kashmir
c) Silk manufacture of Bengal
d) Handpaper industries
e) Glass industries, Lac bangles

British experienced 'industrialisation' in the mid-eighteenth century and at the same time, India experienced 'de-industrialisation'.

According to D.R. Gadgil, causes that led to the decline of handicraft industries in India were:

1. Disappearance of Court Culture:

- ✓ Royal courts were the main source of demand for Indian handicrafts.
- ✓ Due to the abolition of royal court, one major source of demand dried up.

2. Alien rule and foreign influences:

- ✓ European demand sabotaged the artistic quality of Indian handicrafts since they introduced new forms and pattern to suit their tastes which were beyond the craftsmen's comprehension.
- ✓ Secondly, the demand for cheaper goods by European tourists led to extensive adulteration of the raw materials and hasty workmanship.

3. Competition from machine-made goods:

- ✓ The invention of power loom in Europe facilitated the process of de-industrialisation in India.
- ✓ The Indian goods could not match the low price and respect for goods bearing foreign trademark.

4. Tariff Policy:

- ✓ England pursued the policy of protection through the imposition of import duties, but for India, it followed the Classical ideology of free trade.
- ✓ Due to the steady growth of Indian cotton industry, the British House of Commons passed a resolution in 1877 to protect the manufacturers in Lancashire.
- ✓ In 1896, the colonial government used a preferential tariff policy on cash – which cut import duty import duty on British cloth by 3.5% and raised the same on Indian cloth.

5. Internal Causes:

- ✓ No efforts were made to explore the markets for products as India's foreign trade was in the hands of foreigners.

- ✓ Indian artists were at the mercy of foreign merchants.
- ✓ Guild organisation in India was weak.
- ✓ India did not possess any class of industrial entrepreneurs.

Effects of De-industrialisation

- Artisans were displaced from tradition occupations and returned to agriculture for their livelihood. This increased the pressure of population on land.
- Created a class of disguised unemployment and under-employment. Over-burdened agriculture could not generate sufficient surplus and the shortage of capital to improve land further reduced the productivity.
- The class of landless labourers emerged.
- The modern industrial sector, which emerged in India in early 20th century, could not generate enough employment to compensate for the loss of traditional handicraft industry.

ECONOMIC PLANNING IN INDIA

- ✓ Sir M. Vishweswaraiah is considered as the architect of Indian economic planning. He played an important role in creating the First Five Year Plan that was implemented in 1951.
- ✓ Main objective of economic planning is to formulate plans that could help utilise the country's resources most efficiently.
- ✓ In the case of India, economic planning is a subject of Concurrent List. The Central and the State Governments have the right to formulate laws.

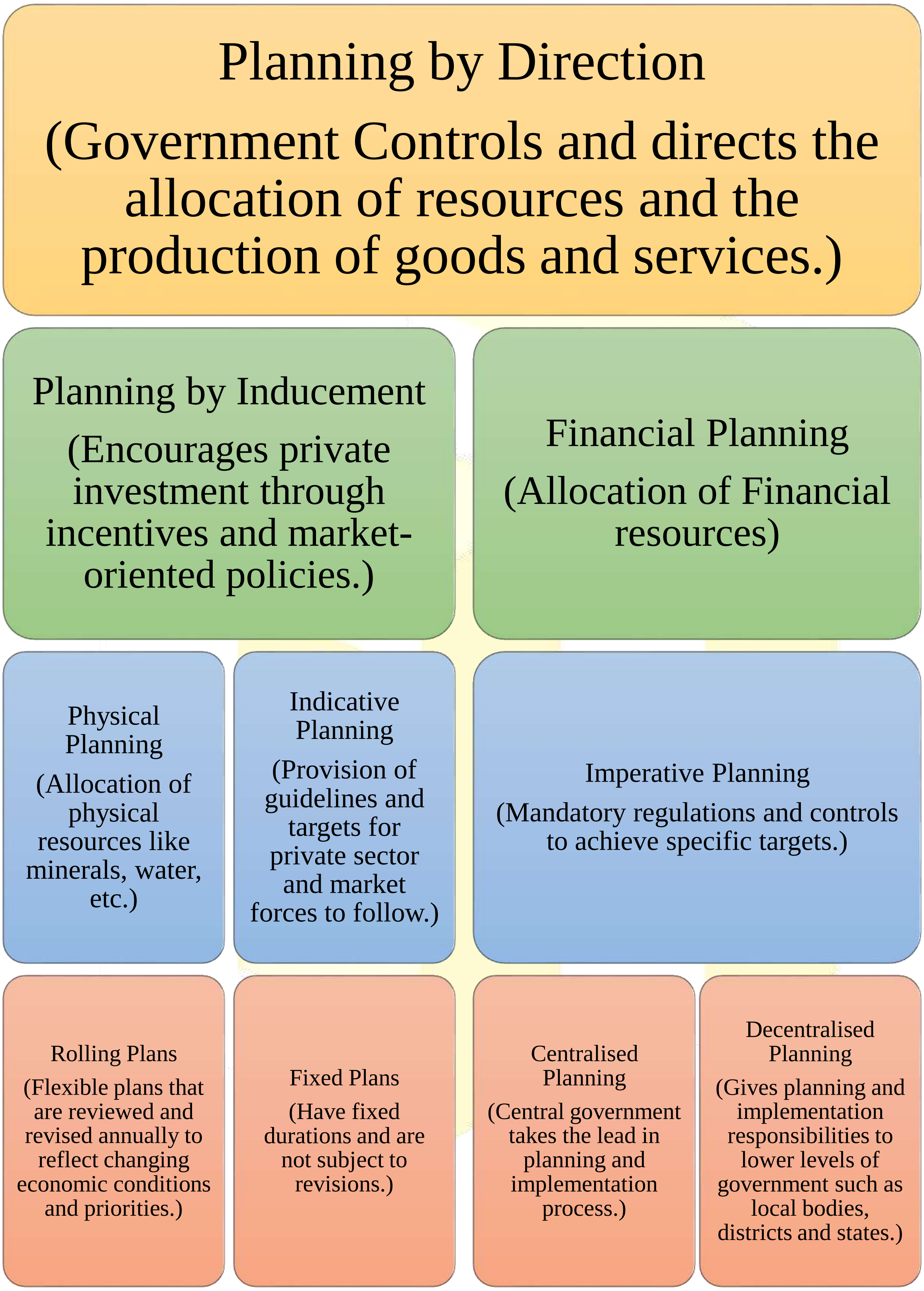
Features of Economic Planning

1. Foster economic growth.
2. Development of modern infrastructure.
3. Improvements in social sector like education and health.
4. Efficient resource allocation.

Objectives of Economic Planning in India

1. Economic growth
2. Poverty reduction
3. Self-sufficiency
4. Removing regional imbalances
5. Promoting social equity and justice
6. Generating employment opportunities especially in the formal sector
7. Increasing standard of living

Types of Economic Planning



UNIT- 5

MATHEMATICS FOR ECONOMICS

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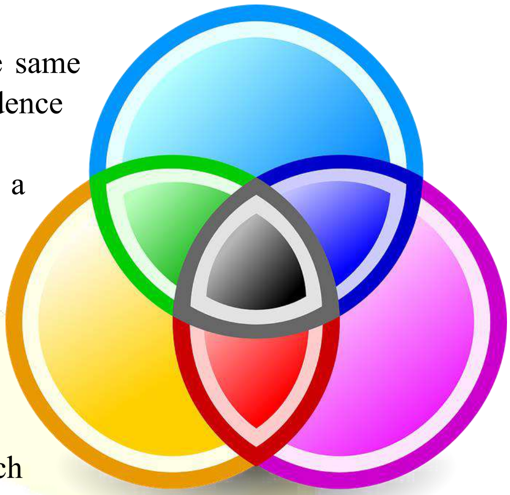
SET THEORY



- ✓ The founder of set theory is Georg Canton.
- ✓ According to him, two sets of elements have the same cardinality, if there is one-to-one correspondence between the sets.
- ✓ A set is a collection of distinct objects forming a group.
- ✓ Each item in a set is called an element of the set.
- ✓ The elements of a set are listed between two curly brackets, that is, {}. For example, a = {a,b,c}. Symbolically, elements are expressed as 'ε'. Example: a ε S.
- ✓ Two sets, say A and B, are considered equal if each element of A is an element of B and each element of B is an element of A. In that case, it is expressed as A = B.
  For example, A = {1,2,3} and B = {3,2,1}, then A = B as order in which the elements are listed holds no significance.
  Similarly, if A = {1,1,2,3} and B = {3,2,1}, even then A = B holds true.
- ✓ Empty set has no element and is denoted as Φ.

Types of sets

1. **Equal Sets**: Two sets A and B are said to be equal sets if and only if element of set A is an element of set B and vice-versa. In other words, the two sets have exactly the same elements, irrespective of their order.
   For example,
   A = {1,4,3} and B = {3,4,1} then A = B
2. **Equivalent Sets**: Two sets A and B are said to be equivalent if for each element in set A there is exactly one element in B and vice-versa.
   For example,
   A = {a, b, c} and B = {3,2,1} then A ≅ B
3. **Subset**: Set A is a subset of set B if and only if each element of set A is an element of set B.
   For example,
   A = {1,2} and B = {1,2,3,4} then A is a subset of B
4. **Disjoint sets**: Two sets A and B are disjoint sets if and only if no element of set A is an element of set B.
   For example,
   A = {1,2,3} and B = {5,6,7} then A and B are disjoint sets.
5. **Universal set**: It is the set which contains all elements under discussion. All the other sets are considered to be the subsets of the universal set. It is denoted by 'U'.
6. **Null set**: It is the set which contains no element. It is denoted by {} or ϕ
7. **Power set**: For a given set A, a power set will be a set of all possible subsets of A. If set A contains 'n' elements, then there will be 2^n subsets of A.

Set Operations

1. Union of sets:

Say there are two sets, A and B. The union of A and B is the set of all those elements that belong to either A or B or both. It is denoted as A∪B

For example: Set A = {1,2} and Set B = {2,3}

Then A∪B = {1, 2, 3}

Laws of Union of Sets:

a. A∪ ϕ = A (Identity law)
b. U∪A = U (Universal law)
c. A∪A = A (Idempotent law)
d. A∪B = B∪A (Commutative law)
e. (A∪B) ∪C = A∪ (B∪C) (Associative law)

2. Intersection of sets:

Say there are two sets, A and B. The intersection of A and B is the set of all those elements that belong to both A and B. It is denoted as A∩B.

For example: Set A = {1,2} and Set B = {2,3}

Then A∩B = {2}

Laws of Intersection of sets:

a. A∩ϕ = ϕ (Identity law)
b. U∩A = A (Universal law)
c. A∩A = A (Idempotent law)
d. A∩B = B∩A (Commutative law)
e. (A∩B)∩C = A∩(B∩C) (Associative law)

3. Difference of sets

Say there are two sets A and B. The difference of sets A and B in this order is the set of all those elements of A which do not belong to B. It is denoted as A-B.

For example: Set A = {1,2,3,4,5} and Set B = {3,5,7,9}

A-B = {1,2,4} and B-A = {7,9}

Symmetric difference of two sets:

Say there are two sets A and B. The symmetric difference of sets A and B is the set (A-B) ∪ (B-A) and is also denoted as AΔB.

For example: Set A = {1,2,3,4} and Set B = {3,4,5,6}

A-B = {1,2} and B-A = {5,6}

AΔB = {1,2,5,6}

Laws of Difference of sets

a. A-B = A∩B'
b. B-A = B∩A'
c, A-B⊆A
d. B-A⊆B
e. A-B = A when A∩B = ϕ
f. (A-B)∪B = A∪B
g. (A-B)∩B = ϕ
h. (A-B)∪(B-A) = (A∪B)-(A∩B)

If there are three sets, then the law of difference are as follows:

a. A-(B∩C) = (A-B)∪(A-C)
b. A-(B∪C) = (A-B)∩(A-C)
c. A∩(B-C) = (A∩B)-(A∩C)
d. A∩(BΔC) = (A∩B)Δ(A∩C)

Notation	Name	The set that consists of
A ∪ B	A union B	The element that belongs to at least one of the sets A and B.
A ∩ B	A intersection B	The elements that belong to both A and B.
A\B (or A-B)	A minus B	The elements that belong to set A but not to B

Let's do a Question

Let A = {1,2,3,4,5} and B = {3,6}. Find A ∪ B, A ∩ B, A\B and B\A.

4. Complement of a set:

Let U be the universal set and A be any subset of U, then complement of A with respect to U is the set of all those elements of U which are not in A and is denoted as A'.

For example: U = {1,2,3,4,5,6} and A = {2,4}

A' = {1,3,5,6}

Properties of Complement of sets

a. (A')' = A = U-A' (Law of double complementation)
b. A∪A' = U
c. A∩A' = ϕ
d. ϕ' = U
e. U' = ϕ
f. (A∪B)' = U-(A∪B)

Laws of Algebra of sets

a. Commutative law:

$$A \cup B = B \cup A$$
And
$$A \cap B = B \cap A$$

b. Distributive law:

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$
And
$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

c. DeMorgan's law

$$(A \cup B)^C = A^C \cap B^C$$
And
$$(A \cap B)^C = A^C \cup B^C$$

Example:

Let A = {1,2,3,4,5} and B = {3,6}. Find A U B, A∩B, A\B and B\A.

1) AUB = {1,2,3,4,5,6}

2) A∩B = {3}

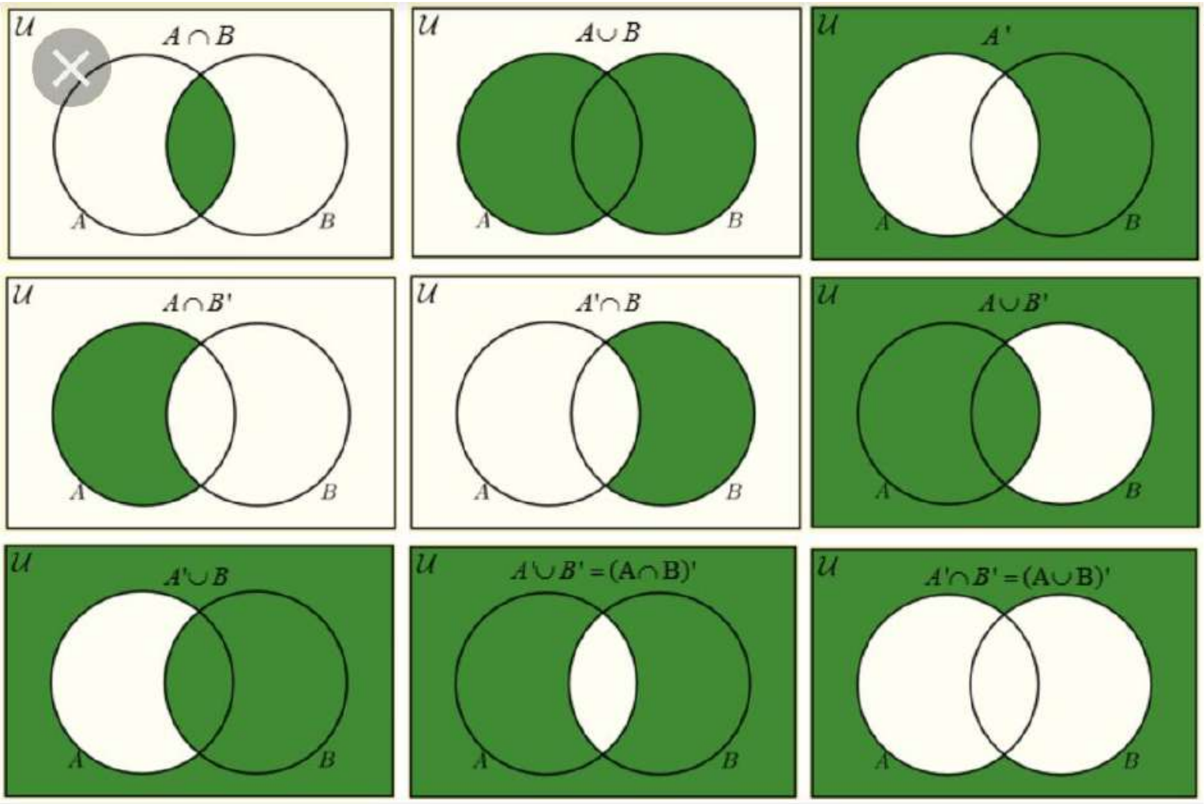
3) A\B = {1,2,4,5}

4) B\A = {6}

- If two sets, A and B, have no element in common, they are said to be disjoint. Therefore, the sets A and B are disjoint if and only if $A \cap B = \Phi$.
- A collection of sets is referred to as the family of sets.
- Universal set is a set that contains the elements of all sets.
- Complement of a set: Difference between universal set and the set in question, that is, $U - A = A^c$. Alternatively, it can be written as $U \backslash A = A^c$.

Venn Diagrams

- ✓ A Venn diagram uses overlapping circles or other shapes to illustrate the logical relationships between two or more sets of items.
- ✓ Popularised by John Venn in the 1880s.



SOME EXERCISES

Question 1: A thousand people took part in a survey to reveal which newspaper, A, B or C, they had read on a certain day. The responses showed that 420 had read A, 316 had read B, and 160 had read C. These figures include 116, who had read both A and B, 100 who had read A and C, and 30 who had read B and C. Finally, all these figures include 16 who had read all three papers.

a) How many had read A, but not B?

b) How many had read C, but neither A nor B?

c) How many had read neither A, B nor C?

(Answer: a) 304, b) 46, c) 334

Question 2: Use Venn Diagrams to prove that:

a) $(A \cup B)^C = A^C \cap B^C$

Propositions

Assertions that are either true or false are called statements or propositions.

Implications

- ✓ Denoted as ⇒ (an implication arrow) and it points in the direction of the logical implications.
  Example: $P \Rightarrow Q$
- ✓ This is read as "P implies Q", "if P, then Q", "Q is a consequence of P".
  Example: $x > 2 \Rightarrow x^2 > 4$.
- ✓ In certain cases where the implication is valid, it may also be possible to draw a logical conclusion in the other direction: $Q \Rightarrow P$. This can be written as:
  $P \Leftrightarrow Q$.
- ✓ The symbol ⇔ is called an equivalence arrow.
  Example: $(x < -2 \text{ or } x > 2) \Leftrightarrow x^2 > 4$.

Necessary and Sufficient Conditions

P is a sufficient condition for Q means: $P \Rightarrow Q$.

Q is a necessary condition for P means: $P \Rightarrow Q$

P is a necessary and sufficient condition for Q means: $P \Leftrightarrow Q$

Try this!

Use appropriate implication or equivalence arrows to represent the following propositions:

a) The equation $2x - 4 = 2$ is fulfilled only when $x = 3$.

b) If $x = 3$, then $2x - 4 = 2$.

c) The equation $x^2 - 2x + 1 = 0$ is satisfied if $x = 1$.

d) If $x^2 > 4$, then $|x|>2$, and conversely.

Answers:
a) $2x - 4 = 2 \Rightarrow x = 3$
b) $x = 3 \Rightarrow 2x - 4 = 2$
c) $x = 1 \Rightarrow x^2 - 2x + 1 = 0$
d) $x^2 > 4 \Leftrightarrow |x| > 2$

NUMBER THEORY

Number theory is the study of the set of positive whole numbers usually called the set of natural numbers.

Classification of Number Theory

1. **Odd Numbers:** Numbers that are not divisible by two. '1' is the first positive odd number.
2. **Even Numbers:** Integers that are divisible by two.
3. **Square Numbers:** Numbers that are multiplied by themselves, give square numbers.
4. **Cube Numbers:** Numbers that are multiplied by themselves three times, give cube numbers.
5. **Prime Numbers:** Prime numbers are numbers that have only two factors, 1 and the number itself.
6. **Composite Numbers:** Numbers that have more than two factors.
7. **Fibonacci Numbers:** A series of numbers where a number is the addition of the last two numbers, starting with 0 and 1. For example, 0,1,1,2,3,5,8,13,21,34,…
8. **Rational Numbers:** Those written in the form of a/b where a and b are both integers. An integer n is also a rational number because n = n/1.
9. **Decimal system:** Also known as 'base 10 system'. It is a positional system with 10 as the base number. Rational numbers that can be written exactly using only a finite number of decimal places are called finite decimal fractions.
10. **Irrational numbers:** Infinitely many new numbers given by the non-periodic decimal fractions. Example: $\sqrt{2}$.

> Please note,
>
> The ratio p/0 is not defined for any real number p.
> This should not be confused with 0/a = 0.

Integer Powers

a^n is called the n^{th} power of a, where a is the base and n is the exponent.

For example, $a^5 = a * a * a * a * a$

Some rules:

- ✓ $a^0 = 1$
- ✓ 0^0 = Undefined
- ✓ $a^{-n} = 1/a^n$

Properties of Powers

1. $a^r * a^s = a^{r+s}$
2. $(a^r)^s = a^{rs}$

3. $a^r \div a^s = a^{r-s}$
4. $(ab)^r = a^r b^r$
5. $(a/b)^r = a^r/b^r = a^r\ b^{-r}$

SOLVE THIS

Simplify these expressions

1) $x^p x^{2p}$

2) $t^s \div t^{s-1}$

3) $a^2 b^3 a^{-1} b^5$

4) $t^p t^{q-1}/t^r t^{s-1}$

Answers:
1) x^{3p} , 2) t, 3) ab^8, 4) $t^{p+q-r-s}$

Compound Interest

A quantity K which increases by p% per year will have increased to

$$K(1+\frac{p}{100})^t$$

After t years. Here, 1+p/100 is called the growth factor for a growth of p%.

A quantity K which decreases by p% per year will have decreased to

$$K(1-\frac{p}{100})^t$$

After t years. Here 1-p/100 is called the growth factor for a decline of p% a year.

Try this:

A new car has been bought for $15,000 and is assumed to decrease in value by 15% per year over a six-year period. What is its value after 6 years?

Answer: $5657

Rules of Algebra

1. a + b = b + a
2. (a + b) + c = a + (b + c)
3. a + 0 = a
4. a + (-a) = 0
5. ab = ba
6. (ab)c = a(bc)
7. 1*a = a
8. $aa^{-1} = 1$ for $a \neq 0$
9. (-a)b = a(-b) = -ab
10. (-a)(-b) = ab
11. a(b+c) = ab + ac
12. (a+b) c = ac + bc

Quadratic Identities

1. $(a+b)^2 = a^2 + 2ab + b^2$
2. $(a-b)^2 = a^2 – 2ab + b^2$
3. $a^2 – b^2 = (a+b)(a-b)$

Expand the following expressions:

1) $(3x+2y)^2$

2) $(1-2z)^2$

3) $(4p+5q)(4p-5q)$

Answers:
1) $9x^2+12xy+4y^2$, 2) $1-4z+4z^2$, 3) $16p^2-25q^2$

Factoring

To factor an expression means to express it as a product of simpler factors.

For example, $5x^2 + 15x = 5x(x+3)$

Try this: Factor the following expressions

1) $16a^2 – 1 =$

2) $x^2y^2 – 25z^2 =$

3) $4u^2 + 8u + 4 =$

Answers:
1) $(4a+1)(4a-1)$, 2) $(xy + 5z)(xy-5z)$, 3) $4(u+1)^2$

Fractions

Properties:

1. $\frac{a.c}{b.c} = \frac{a}{b}$
2. $\frac{-a}{-b} = \frac{a}{b}$
3. $-\frac{a}{b} = \frac{(-1)a}{b} = \frac{-a}{b}$
4. $\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$
5. $\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$
6. $a + \frac{b}{c} = \frac{ac+b}{c}$
7. $a * \frac{b}{c} = \frac{ab}{c}$
8. $\frac{a}{b} * \frac{c}{d} = \frac{ac}{bd}$
9. $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} * \frac{d}{c} = \frac{ad}{bc}$

Simplify these expressions:

1. $\frac{5x^2yz^3}{25xy^2z}$

2. $\frac{x^2+xy}{x^2-y^2}$

3. $\frac{4-4a+a^2}{a^2-4}$

Answers:
1) $\frac{xz^2}{5y}$, 2) $\frac{x}{x-y}$, 3) $\frac{a-2}{a+2}$

Fractional Powers

- ✓ $a^{1/2} = \sqrt{a} \Rightarrow a^{\frac{1}{n}} = \sqrt[n]{a}$
- ✓ $\sqrt{ab} = \sqrt{a}\sqrt{b}$
- ✓ $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$
- ✓ $(\sqrt[n]{a})^n = a$

Compute the following:

1. $16^{3/2}$ =
2. $16^{-1.25}$ =
3. $(1/27)^{-2/3}$ =

Answers:
1) 64, 2) 1/32, 3) 9

Intervals and Absolute Values

Name	Notation	Consists of all x satisfying
The open interval from a to b	(a,b)	$a < x < b$
The closed interval from a to b	[a,b]	$a \le x \le b$
A half-open interval from a to b	(a,b]	$a < x \le b$
A half-open interval from a to b	[a,b)	$a \le x < b$

Absolute Value

The absolute value of the number a is the number |a| defined by

$$|a| = \begin{cases} a, if\ a \ge 0 \\ -a, if\ a < 0 \end{cases}$$

The distance between x_1 and x_2 on the number line is

$|x_1 - x_2| = |x_2 - x_1|$

Summation

Rules for summation

$$\sum_{i=1}^{n}(a_i + b_i) = \sum_{i=1}^{n} a_i + \sum_{i=1}^{n} b_i$$

And

$$\sum_{i=1}^{n} ca_i = c\sum_{i=1}^{n} a_i$$

Newton's Binomial Formula

$$(a+b)^m = a^m + \binom{m}{1}a^{m-1}b + \cdots + \binom{m}{m-1}ab^{m-1} + \binom{m}{m}b^m$$

Where:

$$\binom{m}{k} = \frac{m(m-1)\dots(m-k+1)}{k!}$$

Double Sums

In a finite double sum, the order of summation is immaterial. Summation limits for i and j are independent of each other.

$$\sum_{i=1}^{m}\sum_{j=1}^{n} a_{ij}$$

Example:

Compute: $\sum_{i=1}^{3}\sum_{j=1}^{4}(i+2j)$

= $\sum_{i=1}^{3}[(i+2)+(i+4)+(i+6)+(i+8)]$

= $\sum_{i=1}^{3}(4i+20) = 24+28+32 = \ 84$